HYPER VERSION

Received: *November 4, 1999,* Accepted: *December 7, 1999*

Two-point functions of chiral operators in $\mathcal{N} = 4$ SYM at order g^4

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Abstract: We compute two-point functions of chiral operators $\mathrm{Tr}\Phi^3$ in $\mathcal{N} = 4$ SU(N) supersymmetric Yang-Mills theory to the order g^4 in perturbation theory. We perform explicit calculations using $\mathcal{N} = 1$ superspace techniques and find that perturbative corrections to the correlators vanish for all N. While at order g^2 the cancellations can be ascribed to the nonrenormalization theorem valid for correlators of operators in the same multiplet as the stress tensor, at order g^4 this argument no longer applies and the actual cancellation occurs in a highly nontrivial way. Our result is obtained in complete generality, without the need of additional conjectures or assumptions. It gives further support to the belief that such correlators are not renormalized to *all* orders in g and to *all* orders in N.

Keywords: Renormalization Regularization and Renormalons, 1/N Expansion, D-branes, Superspaces.

JHEP12(1999)006

Contents

1. Introduction

Recently much evidence has been provided in testing the conjectured equivalence of type IIB superstring theory on anti-de-Sitter space (AdS) times a compact manifold to the $\mathcal{N} = 4$ supersymmetric SU(N) Yang-Mills conformal field theory living on the boundary, in the large-N limit and at large 't Hooft coupling $\lambda = g^2N/4\pi$ (g^2 being the Yang-Mills coupling constant) [1]. According to this correspondence correlation functions of operators in the conformal field theory are mapped to appropriate on-shell amplitudes of superstring theory in the bulk AdS background. $\mathcal{N} = 4$ chiral primary operators $\mathrm{Tr}\Phi^k \equiv \mathrm{Tr}\{\Phi^{\{i_1}(z)\Phi^{i_2}(z)\cdots\Phi^{i_k\}}(z)\}$, in the symmetric, traceless representation of SU(4), play a special role in exploring non-perturbative statements concerning the above mentioned connection. These are local operators of the lowest scaling dimension in a given irreducible representation of the superconformal algebra SU$(2,2|4)$, and belong to short multiplets which are chiral under a $\mathcal{N} = 1$ subalgebra. In the large-N limit they correspond to Kaluza Klein modes in the AdS supergravity sector. In the special case of $k = 2$, two- and three-point correlators are given by their free-field theory values for any finite N. In this case their form, fixed up to a constant by conformal invariance, is protected by a nonrenormalization theorem [2] valid for two- and three-point functions of operators in the same multiplet as the stress tensor.

For any strong-weak coupling duality test it is essential to have quantities that do not acquire radiative corrections as one moves from weak to strong coupling. If an exact computation in the supergravity sector shows agreement with a tree level result in the Yang-Mills sector, then there is an indication of a nonrenormalization theorem at work. This is the case for the three–point correlators $\langle \mathrm{Tr}\Phi^{k_1}\mathrm{Tr}\Phi^{k_2}\mathrm{Tr}\Phi^{k_3}\rangle$ computed in ref. [3] in the large-N limit of $\mathcal{N}=4$ SU(N) Yang-Mills: the strong limit result $\lambda = g^2N/4\pi \gg 1$ obtained using type IIB supergravity was shown to agree with the weak 't Hooft coupling limit $\lambda = g^2N/4\pi \ll 1$ in terms of free fields. According to the AdS/CFT correspondence one concludes that the correlators are independent of λ to leading order in N. A stronger conjecture made in ref. [3] claims that three-point functions might be independent of g for *any* value of N. As emphasized above, for the case $k=2$ nonrenormalization properties have been proven to be enjoyed by two- and three-point functions of chiral operators. For general k there exists evidence of nonrenormalization based on proofs that rely on reasonable assumptions (analyticity in harmonic superspace [4] and validity of a generalized Adler-Bardeen theorem [5]).

Explicit perturbative calculations in the $\mathcal{N}=4$ SU(N) Yang-Mills conformal field theory are a way to confirm the conjectures and add insights into potential larger symmetries of the theory. Important steps along this program have been made in [6, 7, 8, 9]. In particular it has been shown that to order g^2 radiative corrections do not affect the two- and three-point functions of chiral operators [6]. The two-point function calculation has been performed for chiral operators with general k. The result has been obtained showing that the order g^2 contribution is proportional to the one for $k=2$ which indeed satisfies the known nonrenormalization theorem mentioned above. The method cannot be extended to higher orders. In this paper we address the non trivial test left open at order g^4. We find that corrections indeed vanish for *all* values of N, then supporting the stronger conjecture of ref. [3].

The paper is organized as follows: in section 2 we briefly illustrate the $\mathcal{N}=4$ theory and give the relevant rules for calculating in $\mathcal{N}=1$ superspace. We present methods of performing perturbation theory calculations which make higher-loop correlators tractable. In section 3 we test our approach computing the order g^4 contributions to the two-point function with $k=2$ and check that their sum vanishes as it should. This offers us the opportunity to give details on the procedure, to collect results that are useful for the subsequent calculation, to emphasize the points of interest on which we will focus in the central part of the work. This is presented in the following section where the two-point correlator with $k=3$ is considered and the order g^4 contributions are collected. The appendices A and B contain a complete list of our notations and details of the actual calculation.

2. The general set up

The physical particle content of $\mathcal{N}=4$ supersymmetric Yang-Mills theory is given by one spin-1 Yang-Mills vector, four spin-1/2 Majorana spinors and six spin-0 particles in the **6** of the R-symmetry group SU(4). All particles are massless and transform under the adjoint representation of the SU(N) gauge group.

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Perturbative calculations are quite difficult to handle using a component field formulation of the theory. (Note that in ref. [6] a component approach was used, but the order g^2 result for the two- and three-point correlators was obtained using a general argumentation based on colour combinatorics. Only a schematic knowledge of the structure of the component action was required.) In general, in order to resum Feynman diagrams at higher-loop orders it is greatly advantageous to work in superspace.

In $\mathcal{N} = 1$ superspace the action can be written in terms of one real vector superfield V and three chiral superfields Φ^i containing the six scalars organized into the $\mathbf{3} \times \bar{\mathbf{3}}$ of SU(3) $\subset$ SU(4) (we follow the notations in [10])

$$
\begin{aligned}
S[J, \bar{J}] = &\int d^8z \ \mathrm{Tr}\left(e^{-gV}\bar{\Phi}_i e^{gV}\Phi^i\right) + \frac{1}{2g^2}\int d^6z \ \mathrm{Tr}W^\alpha W_\alpha + \\
&+\frac{ig}{3!}\mathrm{Tr}\int d^6z \ \epsilon_{ijk}\Phi^i[\Phi^j, \Phi^k] + \frac{ig}{3!}\mathrm{Tr}\int d^6\bar{z} \ \epsilon_{ijk}\bar{\Phi}^i[\bar{\Phi}^j, \bar{\Phi}^k] + \\
&+\int d^6z \ J\mathcal{O} + \int d^6\bar{z} \ \bar{J}\bar{\mathcal{O}},
\end{aligned}
\tag{2.1}
$$

where $W_\alpha = i\bar{D}^2(e^{-gV}D_\alpha e^{gV})$, and $V = V^aT^a$, $\Phi_i = \Phi_i^aT^a$, T^a being SU(N) matrices in the fundamental representation (see appendix A for a list of results on colour structures). We have added to the classical action source terms for the chiral primary operators generically denoted by $\mathcal{O}$ since our goal is the computation of their correlators.

Although in (2.1) the $\mathcal{N} = 4$ supersymmetry invariance is realized only non linearly, the main advantage offered by a $\mathcal{N} = 1$ formulation of the theory resides in the fact that a straightforward off-shell quantum formulation is available. Thus if the aim is to perform higher-loop perturbative calculations this is the most suited approach to follow. Feynman rules are by now standard and we have listed them in appendix B.

We will now focus on the two-point super-correlator for the operator $\mathcal{O}$ = $\mathrm{Tr}(\Phi^{\{i}\Phi^j\Phi^{k\}})$. As in ref. [6], we consider the SU(3) highest weight Φ^1 field and compute $\langle \mathrm{Tr}(\Phi^1)^3 \mathrm{Tr}(\bar{\Phi}^1)^3\rangle$. This is not a restrictive choice since all the other primary chiral correlators can be obtained from this one by SU(3) transformations. What we gain is that we have no flavour combinatorics and we are left to deal with the colour combinatorics only.

We work in euclidean space, with the generating functional defined as

$$
W[J, \bar{J}] = \int \mathcal{D}\Phi \ \mathcal{D}\bar{\Phi}\, \mathcal{D}V \ e^{S[J,\bar{J}]}. \tag{2.2}
$$

Thus the two-point function is given by

$$
\langle \mathrm{Tr}(\Phi^1)^3(z_1)\mathrm{Tr}(\bar{\Phi}^1)^3(z_2)\rangle = \left.\frac{\delta^2 W}{\delta J(z_1)\delta \bar{J}(z_2)}\right|_{J=\bar{J}=0}, \tag{2.3}
$$

where $z \equiv (x, \theta, \bar{\theta})$. We use perturbation theory to evaluate the contributions to $W[J, \bar{J}]$ which are quadratic in the sources, i.e.

$$W[J, \bar{J}] \to \int d^4x_1 \; d^4x_2 \; d^4\theta \; J(x_1, \theta, \bar{\theta}) \frac{F(g^2, N)}{(x_1 - x_2)^6} \bar{J}(x_2, \theta, \bar{\theta}) \,, \tag{2.4}$$

where the x-dependence of the result is fixed by the conformal invariance of the theory, and $F(g^2, N)$ is the function that we want to determine up to order g^4. We will find a result valid for any N.

In order to perform the calculation we have found it convenient to work in momentum space, using dimensional regularization and minimal subtraction scheme. In n dimensions, with $n = 4 - 2\epsilon$, the Fourier transform of the power factor $(x_1 - x_2)^{-6}$ in (2.4) is given by (see (B.3))

$$\frac{1}{(x^2)^3} = \frac{\pi^{-2+\epsilon}}{64} \frac{\Gamma(-1-\epsilon)}{\Gamma(3)} \int d^n p \; \frac{e^{-ipx}}{(p^2)^{-1-\epsilon}} \,. \tag{2.5}$$

The presence of the singular factor $\Gamma(-1-\epsilon) \sim 1/\epsilon$ signals, in momentum space and in dimensional regularization, the UV divergence of the correlation function in (2.4) associated to the short-distance behaviour for $x_1 \sim x_2$. It follows that performing perturbative calculations in momentum space it is sufficient to look for all the contributions to (2.4) that behave like $1/\epsilon$, therefore disregarding finite contributions. In fact, once the divergent terms are determined at a given order in g, using (2.5) one can reconstruct an x-space structure as in (2.4) with a non-vanishing contribution to $F(g^2, N)$. Finite contributions in momentum space would correspond in x-space to terms proportional to ϵ which give rise only to contact terms [11].

The one stated above is the basic rule of our strategy that we can summarize as follows:

- consider all the two-point diagrams from $W[J, \bar{J}]$ with J and $\bar{J}$ on the external legs,
- evaluate all factors coming from combinatorics of the diagram and compute the colour structure using the formulae collected in appendix A,
- perform the superspace D-algebra following standard techniques,
- reduce the result to a multi-loop momentum integral,
- compute its $1/\epsilon$ divergent contribution.

This last step, i.e. the calculation of the divergent part of the various integrals we have achieved using the method of uniqueness [12] and various rules and identities [11, 13] that we have collected in appendix B. Since the theory is at its conformal point, it is not affected by IR divergences. Therefore, even if we work in a massless regularization scheme, we never worry about the IR behavior of our integrals. Moreover, since the theory is finite, the diagrams that we consider do not possess UV divergent subdiagrams. Finally, as a general remark we observe that gauge-fixing the classical action requires the introduction of corresponding Yang-Mills ghosts. However they only couple to the vector multiplet and do not enter our specific calculation.

In the next section, in order to get familiar with the formalism and to check its validity, we present the order g^4 calculation of the two-point correlator with $k = 2$, a non trivial test of our techniques.

3. Preliminary test: $\left\langle \mathrm{Tr}(\Phi^1)^2 \mathrm{Tr}(\bar{\Phi}^1)^2 \right\rangle$ to order g^4

The two-point correlator we are interested in is obtained from $W[J, \bar{J}]$ inserting in the action (2.1) the chiral operators $\mathcal{O} = \mathrm{Tr}(\Phi^1)^2$ and $\bar{\mathcal{O}} = \mathrm{Tr}(\bar{\Phi}^1)^2$. As outlined in the previous section, the relevant contribution is obtained from the generating functional isolating terms of the form

$$W[J, \bar{J}] \to \int d^4x_1\, d^4x_2\, d^4\theta\, J(x_1, \theta, \bar{\theta}) \frac{E(g^2, N)}{(x_1 - x_2)^4} \bar{J}(x_2, \theta, \bar{\theta})\,. \tag{3.1}$$

The general form of (3.1) is fixed by conformal invariance, while the function $E(g^2, N)$ is the unknown to be determined. Fourier transforming from x-space to momentum space

$$\frac{1}{(x^2)^2} = \frac{\pi^{-2+\epsilon}}{16} \frac{\Gamma(-\epsilon)}{\Gamma(2)} \int d^n p\, \frac{e^{-ipx}}{(p^2)^{-\epsilon}} \tag{3.2}$$

makes it clear that non-trivial contributions to the generating functional are given by the divergent part of our Feynman diagrams.

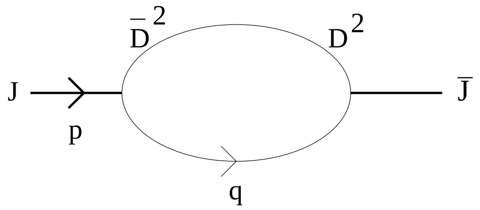

Figure 1: Tree-level contribution to $\langle \mathrm{Tr}(\Phi^1)^2 \mathrm{Tr}(\bar{\Phi}^1)^2 \rangle$.

To start with we consider the tree-level contribution corresponding to the graph in figure 1. The D-algebra in this case is trivial and the one-loop momentum integral gives (see (B.4))

$$\int \frac{d^n q}{q^2 (p-q)^2} = \frac{\Gamma(\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(2-2\epsilon)} \frac{1}{(p^2)^\epsilon} \to \frac{1}{\epsilon}\,. \tag{3.3}$$

Performing the trace operation on the colour indices and inserting factors from combinatorics and Fourier transformations, we obtain

$$\text{figure 1} \to \frac{1}{(4\pi)^2} 2(N^2 - 1) \frac{1}{\epsilon} \int d^4p\, d^4\theta\, J(-p, \theta, \bar{\theta}) \bar{J}(p, \theta, \bar{\theta})\,. \tag{3.4}$$

The order g^2 contribution, once evaluated in superspace gives immediately a zero result: the diagrams one would need to consider are shown in figure 2. Diagram $2a$ does not contribute since the one-loop correction to the chiral propagator vanishes due to a complete cancellation between vector and chiral loops [14]. Diagram $2b$, after completion of the D-algebra leads to a finite momentum integral (see eq. (B.5)).

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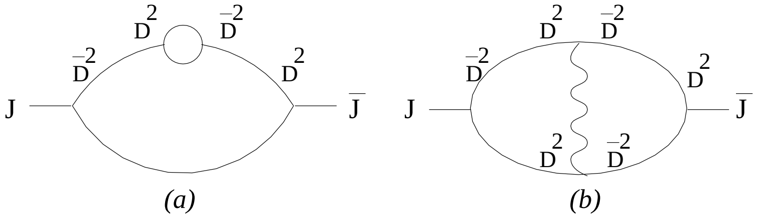

Figure 2: g^2-order contribution to $\langle \mathrm{Tr}(\Phi^1)^2 \mathrm{Tr}(\bar{\Phi}^1)^2 \rangle$.

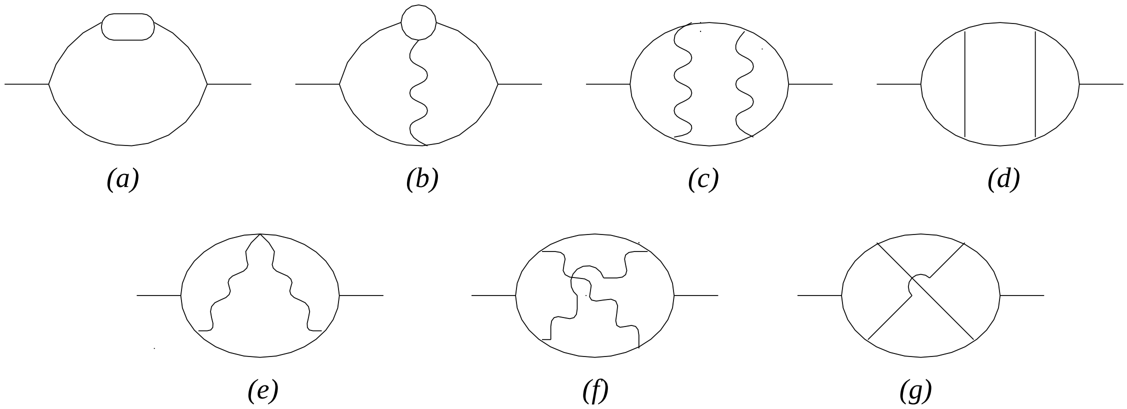

Figure 3: g^4-order contribution to $\langle \mathrm{Tr}(\Phi^1)^2 \mathrm{Tr}(\bar{\Phi}^1)^2 \rangle$.

Now we consider the order g^4 contributions: they are shown in figure 3. In figure 3a we have the insertion of a two-loop propagator correction [14]

$$-2g^4\, N^2\, k_2\, \bar{\Phi}^i_a(p,\theta,\bar{\theta})\, \Phi^i_a(-p,\theta,\bar{\theta})\, p^2 \int \frac{d^n q\ d^n k}{k^2 q^2 (k-q)^2 (k-p)^2 (p-q)^2} =$$
$$= -2g^4\, N^2\, k_2\, \bar{\Phi}^i_a(p,\theta,\bar{\theta})\, \Phi^i_a(-p,\theta,\bar{\theta})\, \frac{1}{(p^2)^{2\epsilon}} [6\zeta(3) + \mathcal{O}(\epsilon)]\,. \qquad (3.5)$$

The two-loop integral has been evaluated using (B.5).

In figure 3b a one-loop vertex correction appears: it corresponds to the effective vertex [14]

$$\frac{g^3}{4} N\, k_2\, i f^{abc}\, \bar{\Phi}^i_a(q,\theta,\bar{\theta}) \Phi^i_b(-p,\theta,\bar{\theta}) \left(4D^\alpha \bar{D}^2 D_\alpha + (p+q)^{\alpha\dot{\alpha}} [D_\alpha, \bar{D}_{\dot{\alpha}}]\right) \times$$
$$\times\, V_c(p-q,\theta,\bar{\theta}) \int \frac{d^n k}{k^2 (k-p)^2 (k-q)^2}\,. \qquad (3.6)$$

Note that a diagram with a vector propagator corrected at order g^2 is absent since at one-loop order there is a complete cancellation among chiral, vector and ghost contributions [14].

A straightforward computation of the D-algebra for the diagrams in figures 3c, 3d and 3e allows to conclude that the corresponding momentum integrals are actually all finite and, as previously observed, not relevant for our purpose. More precisely we have for the diagrams in figures $3c$ and $3d$

$$p^4 \int \frac{d^n k\, d^n q\, d^n r}{k^2 q^2 r^2 (k-q)^2 (q-r)^2 (p-k)^2 (p-q)^2 (p-r)^2} \tag{3.7}$$

and for the diagram in figure $3e$

$$p^2 \int \frac{d^n k\, d^n q\, d^n r}{k^2 q^2 (p-k)^2 (p-r)^2 (p-q)^2 (k-r)^2 (r-q)^2} \,. \tag{3.8}$$

The above integrals are finite by power counting. In general we can disregard all the terms that in the course of the D-algebra end up with spinor derivatives D or $\bar{D}$ acting on the external legs: the resulting momentum integrals are finite and not interesting. Keeping this rule in mind the evaluation of the remaining diagrams is greatly simplified.

The graph in figure $3a$ is easy to compute. Using the result in (3.5) and the one-loop integrals listed in appendix B, with an overall factor

$$16 \frac{1}{(4\pi)^6} g^4\, N^2 (N^2-1) \int d^4 p\, d^4\theta\, J(-p,\theta,\bar{\theta}) \bar{J}(p,\theta,\bar{\theta}) \tag{3.9}$$

one obtains the following divergent contribution

$$\begin{aligned} \text{figure } 3a \to\; & 3\zeta(3) \int \frac{d^n q}{(p-q)^2 (q^2)^{1+2\epsilon}} \\ \to\; & \zeta(3) \frac{1}{\epsilon} \,. \end{aligned} \tag{3.10}$$

From figure $3b$ we obtain a contribution only from the $D^\alpha \bar{D}^2 D_\alpha$ term in the vertex (3.6), with the same overall factor as in (3.9)

$$\begin{aligned} \text{figure } 3b \to & -\int \frac{d^n k\, d^n q\, d^n r}{k^2 (k-q)^2 (k-r)^2 q^2 (q-r)^2} \frac{1}{(p-r)^2} \\ \to & -6\zeta(3) \int \frac{d^n r}{(p-r)^2 (r^2)^{1+2\epsilon}} \\ \to & -2\zeta(3) \frac{1}{\epsilon} \,, \end{aligned} \tag{3.11}$$

where we have used (B.5) and (B.4).

In the same way one analyzes the graph in figure $3f$: after completion of the D-algebra one is left with a divergent integral as the one in (3.11). Factoring out the same overall quantity we have

$$\text{figure } 3f \;\to\; \frac{1}{2} \zeta(3) \frac{1}{\epsilon} \,. \tag{3.12}$$

Exactly the same result is obtained for the last diagram drawn in figure $3g$

$$\text{figure } 3g \;\to\; \frac{1}{2} \zeta(3) \frac{1}{\epsilon} \,, \tag{3.13}$$

It is a trivial matter to sum up the contributions listed in (3.10), (3.11), (3.12) and (3.13) and obtain a vanishing result, as expected from the nonrenormalization theorem.

Before closing this section we note that the diagrams in figures $3f$ and $3g$ lead to planar contributions (i.e. with exactly the same N dependence from colour combinatorics as the other diagrams): indeed to this order nonplanar diagrams are absent. In the next section we will be confronted with a more complicated situation.

4. The main calculation: $\left\langle \mathrm{Tr}(\Phi^1)^3 \mathrm{Tr}(\bar{\Phi}^1)^3 \right\rangle$ to order g^4

Now we present the computation of the two-point function for the chiral operator $\mathcal{O} = \mathrm{Tr}(\Phi^1)^3$. To this end we go back to (2.4) and compute the perturbative contributions to the function $F(g^2, N)$. As previously emphasized, making use of (2.5) we write Feynman diagrams in momentum space and isolate the $1/\epsilon$ poles.

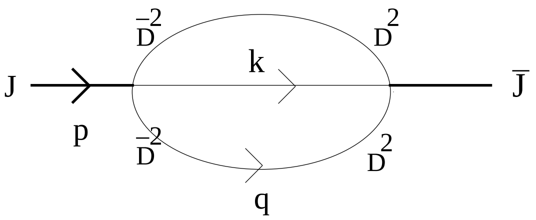

Figure 4: Tree-level contribution to $\langle \mathrm{Tr}(\Phi^1)^3 \mathrm{Tr}(\bar{\Phi}^1)^3 \rangle$.

In figure 4 we have drawn the tree-level contribution. The colour combinatorics is evaluated with the help of (A.13). With an overall factor

$$\frac{3}{(4\pi)^4} \frac{(N^2-1)(N^2-4)}{N} \int d^4p\, d^4\theta\, J(-p,\theta,\bar{\theta}) \bar{J}(p,\theta,\bar{\theta}) \tag{4.1}$$

we obtain

$$\begin{aligned} \text{figure 4} &\to \int \frac{d^n q\, d^n k}{q^2 k^2 (p-q-k)^2} \\ &\to -\frac{1}{4\epsilon} p^2 \,. \end{aligned} \tag{4.2}$$

The result in x-space is readily recovered using (2.5).

The superspace diagrams that enter the order g^2 computation are shown in figure 5. They are nothing else than the ones that appear in figure 2 with one line added from the chiral external vertices. One proves that their contributions vanish with exactly the same reasoning outlined in the previous section. As found in ref. [6] to order g^2 the vanishing of the correlator is due to the fact that it is proportional to the correlator of $\mathcal{O} = \mathrm{Tr}(\Phi^1)^2$ for which the nonrenormalization theorem is valid. However, this is no longer true at order g^4 to which we turn now.

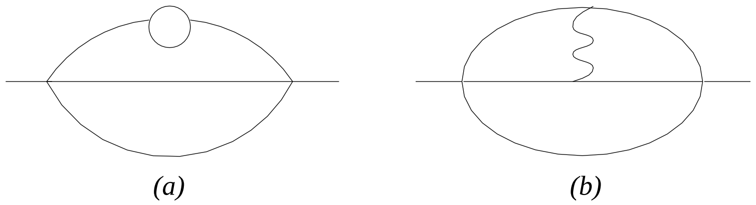

Figure 5: g^2-order contribution to $\langle \mathrm{Tr}(\Phi^1)^3 \mathrm{Tr}(\bar{\Phi}^1)^3 \rangle$.

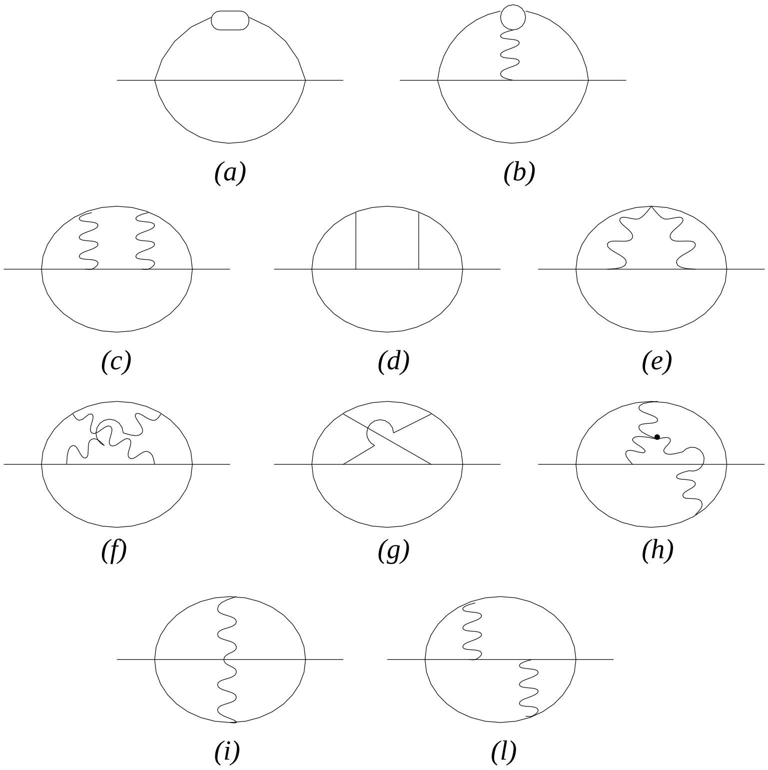

Figure 6: g^4-order contribution to $\langle \mathrm{Tr}(\Phi^1)^3 \mathrm{Tr}(\bar{\Phi}^1)^3 \rangle$.

The diagrams contributing to g^4-order are collected in figure 6. The ones in figure 6*a*–6*g* are the same as in figure 3 with one extra line added from the chiral external vertices. From the result obtained in the previous case at order g^4 (see section 3), we would be tempted to believe that these diagrams still sum up to zero. However this would be a wrong conclusion. In fact, what makes things different is that in 6f and 6g the addition of the extra line changes completely the topology of the diagrams which become really *nonplanar*. As a consequence, their colour combinatorics changes and their N-dependence is distinct from the remaining planar diagrams 6*a*– 6*e*. More specifically, in this case it turns out that the nonplanar diagrams 6f and 6g lead to a vanishing colour combinatorics factor (the details of the calculation are given in appendix A).

The evaluation of the colour coefficient for the other nonplanar diagram in figure 6h reveals again a vanishing contribution (see appendix A for details). The fact that the nonplanar diagrams do not contribute indicates that the final answer is going to be valid for all values of N, independently of any large-N limit. In light of this result it becomes challenging to prove the cancellation of nonplanar diagrams to all orders in the Yang-Mills coupling.

Going back to figure 6, one easily convinces oneself that for the graphs in figures 6c, 6d and 6e the same analysis as in the previous section applies. In this case the addition of the chiral line simply adds a $D^2\bar{D}^2$ factor which accounts for the

D-algebra of one added loop; performing the D-algebra in the diagrams one is left with finite integrals. More precisely we obtain for the diagrams in figures $6c$, $6d$

$$A \int d^n s \frac{s^4}{(p-s)^2 (s^2)^{2+3\epsilon}} \tag{4.3}$$

and for the diagram in figure $6e$

$$B \int d^n s \frac{s^2}{(p-s)^2 (s^2)^{1+3\epsilon}}\,, \tag{4.4}$$

where A and B are finite constants, as a consequence of the results in (3.7) and (3.8). The final s-integration can be performed using (B.8) and a complete finite result is obtained in both cases (4.3) and (4.4). Thus these terms are not relevant and we dismiss them. We note that at this order diagrams containing the scalar superpotential vertex $\epsilon_{ijk}\mathrm{Tr}(\Phi^i[\Phi^j,\Phi^k])$ do not contribute.

We are left with the contributions from figures $6a$, $6b$, $6i$ and $6j$ that we analyze one by one. We will find that a highly nontrivial cancellation occurs.

For every diagram we need compute the specific combinatorics, the various factors from vertices and propagators and the colour structure. Then we have to perform the D-algebra in the loops and finally evaluate the momentum integrals. We factorize for each contribution the same quantity

$$\frac{9}{(4\pi)^8} g^4 \; N(N^2-4)(N^2-1) \int d^4p\, d^4\theta \, J(-p,\theta,\bar\theta)\bar J(p,\theta,\bar\theta)\,. \tag{4.5}$$

The diagram in figure $6a$ which contains the two-loop propagator correction in (3.5) is evaluated directly, using (B.4) for the two successive one-loop integrations,

$$\begin{aligned} \text{figure } 6a \to\; & 12\zeta(3) \int \frac{d^n k\, d^n q}{k^2 (q^2)^{1+2\epsilon}(k+q-p)^2} \\ \to\; & -\frac{3}{2}\zeta(3)\frac{1}{\epsilon}\,p^2\,. \end{aligned} \tag{4.6}$$

The diagram in figure $6b$ contains the one-loop vertex correction given in (3.6). In this case too, after completion of the D-algebra,one obtains a relevant contribution only from the term in (3.6) which has the $D^\alpha \bar D^2 D_\alpha$ factor. The resulting momentum integral is

$$\text{figure } 6b \;\to\; -4\int \frac{d^n k\, d^n q\, d^n r\, d^n l}{k^2(k-q)^2(r-q)^2(p-r)^2 l^2 (l-k)^2 (l-q)^2}\,. \tag{4.7}$$

First using (B.5) one performs the k and l integrations, then with the help of (B.4) one evaluates the integrals on the q and the r variables

$$\text{figure } 6b \;\to\; 3\zeta(3)\frac{1}{\epsilon}\,p^2\,. \tag{4.8}$$

In the same way for the graph in figure $6i$ one has (see formula (B.7))

$$\begin{aligned} \text{figure } 6i &\to -p^2 \int \frac{d^n k\, d^n q\, d^n r\, d^n s}{k^2 q^2 (k-q)^2 (p-k-r)^2 (p-q-s)^2 (r-s)^2 r^2 s^2} \\ &\to -5\zeta(5) \frac{1}{\epsilon}\, p^2 \, . \end{aligned} \tag{4.9}$$

Finally we concentrate on the evaluation of the diagram in Fig. $6j$ which after D-algebra gives rise to the following momentum integral

$$\text{figure } 6j \;\to\; 2 \int \frac{d^n k\, d^n q\, d^n r\, d^n s \quad (p-r)^2 (p-q)^2}{k^2 (k-q)^2 q^2 r^2 (r-s)^2 s^2 (p-k-r)^2 (p-q-r)^2 (p-q-s)^2} \, . \tag{4.10}$$

The divergent contributions from this integral are obtained expanding the numerator and keeping only terms with at the most p^2-powers. Exploiting the symmetry of the integrand under $r \leftrightarrow q$ and $k \leftrightarrow s$ we replace the numerator as

$$(p-r)^2 (p-q)^2 \;\to\; 2p^2 r^2 + r^2 q^2 - 4r^2 p\cdot q + 4 p\cdot r\, p\cdot q \, . \tag{4.11}$$

We denote the integrals corresponding to the four terms by J_1, J_2, J_3 and J_4 respectively and analyze them one by one. Using (B.10) we obtain

$$J_1 \;\to\; 20\zeta(5) \frac{1}{\epsilon}\, p^2 \, . \tag{4.12}$$

Using (B.9) we obtain

$$J_2 \;\to\; -\frac{3}{2}\zeta(3) \frac{1}{\epsilon}\, p^2 \, . \tag{4.13}$$

Simple manipulations which exploit the symmetries of the integrand allow to reduce the J_3 integral to the one in (B.10), so that we obtain

$$J_3 \;\to\; -10\zeta(5) \frac{1}{\epsilon}\, p^2 \, . \tag{4.14}$$

We are left with the evaluation of J_4 which requires a more complicated reasoning. The integral we are dealing with is

$$J_4 = 8 p^\mu p^\nu \int \frac{d^n k \;\, d^n q \;\, d^n r \;\, d^n s \quad r_\mu q_\nu}{k^2 (k-q)^2 q^2 r^2 (r-s)^2 s^2 (p-k-r)^2 (p-q-r)^2 (p-q-s)^2} \, . \tag{4.15}$$

We observe that the divergent part of the integral is proportional to $\delta_{\mu\nu}$ so that restricting our attention to that part we can write

$$J_4^{\rm div} \;\to\; \frac{8}{n} p^2 \int \frac{d^n k\, d^n q\, d^n r\, d^n s \quad r\cdot q}{k^2 (k-q)^2 q^2 r^2 (r-s)^2 s^2 (p-k-r)^2 (p-q-r)^2 (p-q-s)^2} \, . \tag{4.16}$$

To evaluate the divergent contribution in (4.16) we exploit the result in eq. (B.7) by first rewriting that integral as

$$I_4 = \int \frac{d^n k\, d^n q\, d^n r\, d^n s \quad (p-q-r)^2}{(p-q-r)^2 k^2 q^2 (k-q)^2 (p-k-r)^2 (p-q-s)^2 (r-s)^2 r^2 s^2} \, , \tag{4.17}$$

where we have multiplied and divided by a factor $(p-q-r)^2$. Now we expand the square in the numerator and neglect terms which, by power counting, generate finite integrals. Using the result in (B.10) with a suitable redefinition of the integration variables, we can write

$$\begin{aligned} I_4^{\text{div}} &= \int \frac{d^nk\, d^nq\, d^nr\, d^ns\, q^2+r^2+2r\cdot q}{(p-q-r)^2k^2q^2(k-q)^2(p-k-r)^2(p-q-s)^2(r-s)^2r^2s^2} \\ &= 2\,\tilde{I}_4^{\text{div}} + \left(\frac{4}{n}p^2\right)^{-1} J_4^{\text{div}}\,. \end{aligned} \tag{4.18}$$

On the other hand, from eqs. (B.7) and (B.10) we read

$$I_4^{\text{div}} = 5\zeta(5)\,\frac{1}{\epsilon}\,, \qquad \tilde{I}_4^{\text{div}} = 5\zeta(5)\,\frac{1}{\epsilon} \tag{4.19}$$

Comparing eqs. (4.18) and (4.19) we obtain

$$\begin{aligned} J_4^{\text{div}} &= \frac{4}{n}p^2[I_4^{div} - 2\tilde{I}_4^{\text{div}}] \\ &\to -5\zeta(5)\frac{1}{\epsilon}\,p^2\,. \end{aligned} \tag{4.20}$$

Finally summing the divergences from (4.12), (4.13), (4.14) and (4.20) we have

$$\text{figure } 6j \;\to \left[5\zeta(5) - \frac{3}{2}\zeta(3)\right]\frac{1}{\epsilon}\,p^2\,. \tag{4.21}$$

At this point it is simple to add the four contributions in (4.6), (4.8), (4.9) and (4.21) and check the complete cancellation of the $1/\epsilon$ terms. It is interesting to note that, while the diagrams $6a$, $6b$ only contribute with a divergent term proportional to $\zeta(3)$ and the diagram $6i$ gives only a $\zeta(5)$-term, from the diagram $6j$ both terms arise with the correct coefficients to cancel completely the divergence.

5. Conclusions

We have computed perturbatively up to g^4-order the two point correlation function for the chiral primary operator $\mathrm{Tr}\Phi_1^3$ in $\mathcal{N}=4$ SU(N) SYM theory. We have found a complete cancellation of quantum corrections for any finite N. Our result represents the first $\mathcal{O}(g^4)$ direct check of the nonrenormalization theorem conjectured on the basis of the AdS/CFT correspondence [3]. It supports also the stronger claim [3] that there might be no quantum corrections at all, for any finite N.

We have performed the calculation in $\mathcal{N}=1$ superspace using dimensional regularization. The loop-integrals have been evaluated in momentum space with the method of uniqueness [12, 13]. In momentum space nontrivial, potential contributions appear as local divergent terms that are easily isolated and evaluated. Finite contributions would correspond to contact terms and can be neglected.

Our procedure is applicable to the perturbative analysis of more complicated cases. Two-point functions for $\mathrm{Tr}\Phi^k$, $k>3$, three-point functions and extremal correlators for chiral primary operators are now under investigation.

Acknowledgments

We wish to thank Dan Freedman for his continuous encouragement, enthusiasm and helpful suggestions during the course of this calculation.

This work has been supported by the European Commission TMR programme ERBFMRX-CT96-0045, in which S.P. and D.Z. are associated to the University of Torino.

A. Colour conventions and relevant identities

In this appendix we give our conventions and list a set of colour structure identities that we have used in the course of the calculation. In addition we evaluate explicitly the colour coefficients for the g^4-order nonplanar diagrams in figures $6f$, $6g$ and $6h$.

The SU(N) generators $\{T_a\}$, $a = 1, \ldots, N^2 - 1$, satisfy the algebra

$$[T_a, T_b] = if_{abc}T_c \,, \tag{A.1}$$

where f_{abc} are the structure constants. In the fundamental representation they are given by $N \times N$ traceless matrices satisfying

$$\mathrm{Tr}(T_a T_b) = k_2 \, \delta_{ab} \,. \tag{A.2}$$

Here k_2 is an arbitrary constant (usually $k_2 = 1/2$) which can be eliminated by a suitable rescaling of the T_a-matrices and the structure constants. We leave it unspecified, since in the evaluation of correlation functions it only appears at an intermediate stage, the final answer being independent of k_2.

The anticommutator of two generators defines a totally symmetric tensor d_{abc}, according to the relation

$$\{T_a, T_b\} = 2k_2 \left(\frac{1}{N}\delta_{ab} + d_{abc}T_c \right) . \tag{A.3}$$

Now, using the previous definitions and the relation

$$T^a_{ij}T^a_{kl} = k_2 \left(\delta_{il}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{kl} \right) \tag{A.4}$$

one can easily derive the following identities

$$(T^a T^b T^a)_{ij} = -\frac{k_2}{N}T^b_{ij} \tag{A.5}$$

$$\mathrm{Tr}(T_a T_b T_c) = \frac{k_2}{2}[(2k_2)d_{abc} + if_{abc}] \tag{A.6}$$

$$f_{acd}f_{bcd} = 2k_2 N \delta_{ab} \qquad d_{acd}d_{bcd} = \frac{1}{2k_2}\frac{N^2-4}{N}\delta_{ab} \tag{A.7}$$

$$f_{alm}f_{bmn}f_{cnl} = k_2 N f_{abc} \tag{A.8}$$

$$d_{abc}d_{aem}d_{bdm} = \frac{1}{k_2}\frac{N^2-12}{4N}d_{cde} \tag{A.9}$$

$$\mathrm{Tr}(T_aT_bT_c)d_{abd} = k_2\frac{N^2-4}{2N}\delta_{cd} \tag{A.10}$$

$$\mathrm{Tr}(T_aT_bT_c)f_{amd}f_{bme} = k_2 N\,\mathrm{Tr}(T_cT_dT_e) \tag{A.11}$$

$$\mathrm{Tr}(T_aT_bT_c)d_{amd}d_{bme} = \frac{1}{k_2}\left[\frac{N^2-8}{4N}\mathrm{Tr}(T_dT_eT_c) - \frac{1}{N}\mathrm{Tr}(T_eT_dT_c)\right] \tag{A.12}$$

$$\mathrm{Tr}(T_aT_bT_c)[\mathrm{Tr}(T_aT_bT_c)+\mathrm{Tr}(T_aT_cT_b)] = k_2^3\frac{(N^2-4)(N^2-1)}{N}\,. \tag{A.13}$$

For the product of four structure constants one obtains

$$f_{amn}f_{bnp}f_{cpr}f_{drm} = (2k_2)^2\left[\delta_{ab}\delta_{cd}+\delta_{ad}\delta_{bc}+k_2\frac{N}{2}(d_{abe}d_{cde}-d_{ace}d_{bde}+d_{ade}d_{bce})\right]. \tag{A.14}$$

Finally, from Jacobi identity one derives

$$f_{abm}f_{cdm}+f_{cbm}f_{dam}+f_{dbm}f_{acm} = 0\,. \tag{A.15}$$

Now we concentrate on the evaluation of the colour coefficients for the diagrams $6f$ and $6g$. The colour structure arising from these diagrams is

$$\mathcal{C} \equiv [\mathrm{Tr}(T_aT_bT_c)+\mathrm{Tr}(T_aT_cT_b)]\,f_{amn}f_{enp}f_{bpr}f_{drm}\,[\mathrm{Tr}(T_dT_eT_c)+\mathrm{Tr}(T_dT_cT_e)]\,. \tag{A.16}$$

Using the identities (A.6) and (A.14) the previous expression can be rewritten as

$$\mathcal{C} = 16k_2^6\left[2d_{abc}d_{abc}+k_2\frac{N}{2}\left(2d_{abc}d_{aem}d_{bdm}d_{dec}-d_{abc}d_{abm}d_{edm}d_{dec}\right)\right]. \tag{A.17}$$

We exploit the identities (A.7) and (A.9) for the product of two and three d-tensors respectively, so that we obtain

$$\begin{aligned}\mathcal{C} &= 8k_2^5\left\{\frac{2}{N}(N^2-1)(N^2-4)+\frac{1}{4N}(N^2-1)(N^2-4)\left[(N^2-12)-(N^2-4)\right]\right\}\\ &= 0\,.\end{aligned} \tag{A.18}$$

Finally, we prove the vanishing of the colour coefficient for the nonplanar diagram $6h$. The colour structure for this diagram is

$$[\mathrm{Tr}(T_aT_bT_c)+\mathrm{Tr}(T_aT_cT_b)]\,f_{amd}f_{bne}f_{cpg}f_{mnp}\,[\mathrm{Tr}(T_dT_eT_g)+\mathrm{Tr}(T_dT_gT_e)]\,. \tag{A.19}$$

This expression is trivially zero since the product of the four structure constants is antisymmetric under the exchange $a\leftrightarrow b$ and $d\leftrightarrow e$, whereas the rest of the expression is symmetric. However, a stronger result holds which states the vanishing of (A.19) without exploiting any symmetry of the expression. Let us concentrate, for example, on the first piece

$$\mathrm{Tr}(T_aT_bT_c)f_{amd}f_{bne}f_{cpg}f_{mnp}\,\mathrm{Tr}(T_dT_eT_g)\,. \tag{A.20}$$

By means of the Jacobi identity (A.15) applied to the product $f_{cpg}f_{mnp}$, this expression can be easily rewritten as

$$\begin{aligned}&\mathrm{Tr}(T_aT_bT_c)\,[-f_{bne}f_{cnp}f_{dma}f_{gmp}+f_{amd}f_{cmp}f_{enb}f_{gnp}]\;\mathrm{Tr}(T_dT_eT_g)=\\&\qquad=k_2^2N^2[-\mathrm{Tr}(T_aT_eT_p)\mathrm{Tr}(T_aT_eT_p)+\mathrm{Tr}(T_dT_bT_p)\mathrm{Tr}(T_dT_bT_p)]=0\,,\end{aligned}\tag{A.21}$$

where the identity (A.11) has been used. Analogous cancellations occur for the other three pieces of the sum in (A.19).

B. Conventions and details of the calculation in momentum space

The quantization procedure of the classical action in (2.1) requires the introduction of a gauge fixing and corresponding ghost terms. We have found it convenient to work in Feynman gauge, so that the vector and the chiral superfield propagators are treated on the equal footing. Here we do not repeat the various steps that lead to the construction of the quantum action (see for example [10]); we simply give the essential ingredients. The ghost superfields only couple to the vector multiplet and are not interesting for our calculation. The relevant interactions are the ones in (2.1). In momentum space we have the superfield propagators

$$\langle V^aV^b\rangle=-\frac{1}{k_2}\frac{\delta^{ab}}{p^2}\qquad\langle\Phi_i^a\bar{\Phi}_j^b\rangle=\frac{1}{k_2}\delta_{ij}\frac{\delta^{ab}}{p^2}\,.\tag{B.1}$$

The vertices are read directly from the interaction terms in (2.1), with additional $\bar{D}^2$, D^2 factors for chiral, antichiral lines respectively. The ones that we need are the following

$$\begin{aligned}V_1&=igk_2f_{abc}\delta^{ij}\bar{\Phi}_i^aV^b\Phi_j^c\\V_2&=-\frac{i}{2}gk_2f_{abc}V^a\bar{D}^2D^\alpha V^bD_\alpha V^c\\V_3&=\frac{g^2}{2}k_2\delta^{ij}f_{adm}f_{bcm}V^aV^b\bar{\Phi}_i^c\Phi_j^d\\V_4&=-\frac{g}{3!}\epsilon^{ijk}f_{abc}\Phi_i^a\Phi_j^b\Phi_k^c\qquad\bar{V}_4=-\frac{g}{3!}\epsilon^{ijk}f_{abc}\bar{\Phi}_i^a\bar{\Phi}_j^b\bar{\Phi}_k^c\,.\end{aligned}\tag{B.2}$$

All the calculations are performed in n dimensions with $n=4-2\epsilon$.

We make use of the n-dimensional Fourier transform

$$\int d^np\,\frac{e^{-ipx}}{(p^2)^\nu}=\pi^{n/2}2^{n-2\nu}\frac{\Gamma(n/2-\nu)}{\Gamma(\nu)}\frac{1}{(x^2)^{n/2-\nu}}\,,\qquad\nu\neq\frac{n}{2},\frac{n}{2}+1,\ldots\tag{B.3}$$

and perform the calculation in momentum space. The various integrals we have to deal with are all computed making use of the method of uniqueness [12]. This approach, which consists in an analytical calculation of multiloop Feynman diagrams, is particularly efficient for massless Feynman integrals of a single variable (either momentum or coordinate). Since we are evaluating a two-point correlator with fields that are all massless, we are just there where the method best applies.

At intermediate stages of the calculation we drop 2π factors and reinstate them at the end, multiplying the final result by a $1/(4\pi)^2$ factor for every loop. With this understanding we give some of the integrals that we made use of:
At one loop

$$I_1 = \int \frac{d^n k}{(k^2)^\alpha [(p-k)^2]^\beta} = \frac{\Gamma(\alpha+\beta-n/2)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(n/2-\alpha)\Gamma(n/2-\beta)}{\Gamma(n-\alpha-\beta)} \frac{1}{(p^2)^{\alpha+\beta-n/2}} . \tag{B.4}$$

At two loops

$$I_2 = \int \frac{d^n k \; d^n q}{k^2 q^2 (k-q)^2 (k-p)^2 (q-p)^2} = \frac{1}{(p^2)^{1+2\epsilon}} [6\zeta(3) + \mathcal{O}(\epsilon)] . \tag{B.5}$$

At three loops

$$I_3 = \int \frac{d^n k \; d^n q \; d^n r}{q^2 k^2 (p-q)^2 (r-q)^2 (p-r)^2 (r-k)^2 (k-q)^2} = \frac{1}{(p^2)^{1+3\epsilon}} [20\zeta(5) + \mathcal{O}(\epsilon)] . \tag{B.6}$$

At four loops

$$I_4 = \int \frac{d^n k \; d^n q \; d^n r \; d^n s}{k^2 q^2 (k-q)^2 (p-k-r)^2 (p-q-s)^2 (r-s)^2 r^2 s^2} = \frac{1}{(p^2)^{4\epsilon}} \; \frac{1}{\epsilon} \; [5\zeta(5) + \mathcal{O}(\epsilon)] . \tag{B.7}$$

From the previous integrals we can derive several other results that we used in the course of our calculation: from (B.4) one obtains

$$\tilde{I}_0 = \int \frac{d^n k}{(k^2)^{\alpha\epsilon} (p-k)^2} = -\frac{\alpha}{2(1+\alpha)} p^2 + \mathcal{O}(\epsilon) . \tag{B.8}$$

From (B.5) we have

$$\begin{aligned} \tilde{I}_3 &= \int \frac{d^n k \; d^n q \; d^n s}{k^2 q^2 (k-q)^2 (k-s)^2 (q-s)^2 (p-s)^2} \\ &= [6\zeta(3) + \mathcal{O}(\epsilon)] \int \frac{d^n s}{(s^2)^{1+2\epsilon} (p-s)^2} \\ &= \frac{1}{(p^2)^{3\epsilon}} \frac{1}{\epsilon} [2\zeta(3) + \mathcal{O}(\epsilon)] . \end{aligned} \tag{B.9}$$

In the same way from (B.6) we have

$$\begin{aligned} \tilde{I}_4 &= \int \frac{d^n k \; d^n q \; d^n r \; d^n s}{q^2 k^2 (s-q)^2 (r-q)^2 (s-r)^2 (r-k)^2 (k-q)^2 (p-s)^2} \\ &= [20\zeta(5) + \mathcal{O}(\epsilon)] \int \frac{d^n s}{(s^2)^{1+3\epsilon} (p-s)^2} \\ &= \frac{1}{(p^2)^{4\epsilon}} \frac{1}{\epsilon} [5\zeta(5) + \mathcal{O}(\epsilon)] . \end{aligned} \tag{B.10}$$

References

[1] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200];
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[2] S.S. Gubser and I.R. Klebanov, *Absorption by branes and Schwinger terms in the world volume theory*, *Phys. Lett.* **B 413** (1997) 41 [hep-th/9708005];
D. Anselmi, D.Z. Freedman, M.T. Grisaru and A.A. Johansen, *Nonperturbative formulas for central functions of supersymmetric gauge theories*, *Nucl. Phys.* **B 526** (1998) 543 [hep-th/9708042].

[3] S. Lee, S. Minwalla, M. Rangamani and N. Seiberg, *Three-point functions of chiral operators in $D = 4$, $N = 4$ SYM at large-N*, *Adv. Theor. Math. Phys.* **2** (1998) 697 [hep-th/9806074].

[4] B. Eden, P.S. Howe and P.C. West, *Nilpotent invariants in $N = 4$ sym*, *Phys. Lett.* **B 463** (1999) 19 [hep-th/9905085];
P.S. Howe, C. Schubert, E. Sokatchev and P.C. West, *Explicit construction of nilpotent covariants in $N = 4$ SYM*, hep-th/9910011.

[5] A. Petkou and K. Skenderis, *A non-renormalization theorem for conformal anomalies*, hep-th/9906030.

[6] E. D'Hoker, D.Z. Freedman and W. Skiba, *Field theory tests for correlators in the AdS/CFT correspondence*, *Phys. Rev.* **D 59** (1999) 045008 [hep-th/9807098].

[7] W. Skiba, *Correlators of short multi-trace operators in $N = 4$ supersymmetric Yang-Mills*, *Phys. Rev.* **D 60** (1999) 105038 [hep-th/9907088].

[8] E. D'Hoker, D.Z. Freedman, S.D. Mathur, A. Matusis and L. Rastelli, *Extremal correlators in the AdS/CFT correspondence*, hep-th/9908160.

[9] M. Bianchi and S. Kovacs, *Non-renormalization of extremal correlators in $N = 4$ SYM theory*, *Phys. Lett.* **B 468** (1999) 102 [hep-th/9910016].

[10] S.J. Gates, M.T. Grisaru, M. Roček and W. Siegel, *Superspace*, Benjamin-Cummings, Reading, MA, 1983.

[11] K.G. Chetyrkin and F.V. Tkachev, *Integration by parts: the algorithm to calculate beta functions in 4 loops*, *Nucl. Phys.* **B 192** (1981) 159.

[12] D.I. Kazakov, *The method of uniqueness, a new powerful technique for multiloop calculations*, *Phys. Lett.* **B 133** (1983) 406; *Analytical methods for multiloop calculations: Two lectures on the method of uniqueness*, Dubna preprint JINR-E2-84-410 (1984).

[13] K.G. Chetyrkin, A.L. Kataev and F.V. Tkachev, *New approach to evaluation of multiloop Feynman integrals: the Gegenbauer polynomial X-space technique*, *Nucl. Phys.* **B 174** (1980) 345.

[14] M.T. Grisaru, W. Siegel and M. Rocek, *Improved methods for supergraphs*, *Nucl. Phys.* **B 159** (1979) 429.